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Educational Television Series 1968-69

Teachers' Guide

DEPT. OF EDUCATION
HISTORICAL COLLECTION

Mathematics INTERMEDIATE DIVISION

Adventures in Mathematics

Educational Television Branch
ONTARIO DEPARTMENT OF EDUCATION
1670 Bayview Avenue, Toronto 17

MATHEMATICS
INTERMEDIATE DIVISION
Adventures in Mathematics

7 programs of 10 minutes each for students in the Intermediate Division.

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INTRODUCTION

It is hoped that the students of Grades 7 to 10 will gain a more mature appreciation of mathematics by studying the discoveries of some of our great mathematicians of the past and learn something of the development of mathematical ideas through the years to the present day. It is therefore the primary intention of this series to portray, in a fundamental way, mathematics as a dynamic, creative process.

Each program is designed to be used with approximately one preparation and one follow-up lesson, with the emphasis on the research and discovery approach. It is also suggested that during the viewing of a program the students be allowed to take notes and that these notes be used as a basis for discussion immediately following the program. In addition some of the programs are structured so that the students have the opportunity to make spontaneous responses to questions which are raised in the course of each of the programs.

PROGRAM 1

THE HISTORY OF π

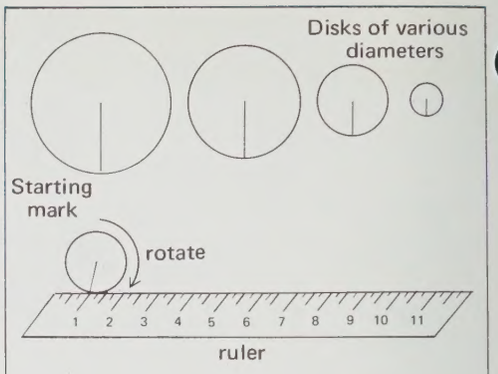
The first program in this series deals with one of the most famous numbers in mathematics, π. It shows how the ancient Greeks, Egyptians and early Biblical scholars were acquainted with this number and how they attempted to determine its value.

PROGRAM CONTENT

For circular objects, man noticed that when the measure of the circumference was divided by the measure of the diameter, the value obtained was the same. Early attempts to obtain the value of π were basically geometrical. Reference is made to the attempts by Antiphon, Bryson and Archimedes to find its value. The reason why the symbol π is used is shown. As mathematicians determine values for π, it is noticed that the decimal approximations to the value of π improve. With the development of higher mathematics, the attempts to find the value of π become more sophisticated. The methods of the French mathematician F. Viète and the Englishman, J. Wallis are described. The German mathematician, G. Leibniz, used a simple expression to find the value of π. A simple experiment is described which enables one to find the approximate value of π. With the aid of computers today, the value of π can be obtained to hundreds of thousands of decimal places.

SUGGESTED PRE-VIEWING ACTIVITIES

1. Review the nature of and the definition for diameter, radius and circumference.
2. The circumference and diameter of various circular objects could be measured and a record kept of the value of the ratio: circumference divided by the diameter.
3. Cut out circular disks of various sizes; measure their diameters and circumferences as illustrated below, then prepare a table.



Diameter	Circum.	$\frac{C}{D}$
3"	9.5"	3.15

SUGGESTED POST-VIEWING ACTIVITIES

1. Examine the contribution of some of the mathematicians mentioned during the program; Antiphon, Archimedes, Viète, Fermat, Euler, Gregory, and Lindemann.
2. Students could find some approximate values of π by using the methods listed below to more and more terms.
   J. Wallis:
   $$\frac{\pi}{2} = \frac{2}{1} \times \frac{2}{3} \times \frac{4}{3} \times \frac{4}{5} \times \frac{6}{5} \times \frac{6}{7} \times \frac{8}{7} \times \frac{8}{9} \cdots$$
   G. Leibniz:
   $$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \cdots$$
3. The following experiment, outlined on the program, could be attempted to find an approximate value for π. Use ten uniform one-inch sticks. On a large piece of paper, mark off parallel lines two inches apart. From a height of about one foot, drop these ten sticks, and record the number of sticks that touch or cross the lines. Repeat this procedure many times.

   Divide the total number of sticks dropped by the total number that touched or crossed the lines, and an approximate value for π can be obtained.

References

EVES, *An Introduction to the History of Mathematics,* Holt Rinehart Winston.

PROGRAM 2

FROM TIME TO TIME

This program attempts to show how man very early in his history strove to keep track of the passage of time. Although this may seem an easy thing to do, it is not, for it is extremely difficult to accurately record or even define time. It is hoped that through this study of the obstacles encountered and overcome in early calendars, our viewers may gain a greater appreciation of what problems are involved in the recording of time.

PROGRAM CONTENT

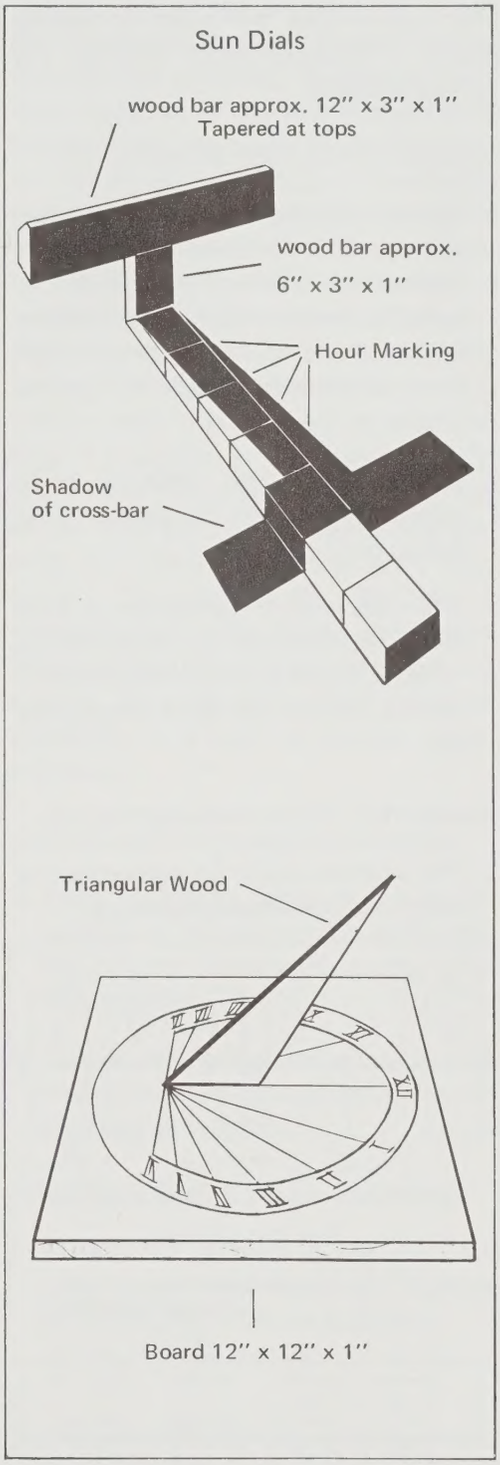
The development of the Gregorian calendar and the attempts of man to bring it into harmony with nature are shown. The early Roman calendar and the revisions of it by Julius Caesar are described. The Romans used only three distinct days of the month; the Kalends, Nones, and Ides, to reckon dates. Two examples are shown: March 2, March 17. The Roman calendar was not accurate and accumulated errors. In 1582, Pope Gregory corrected this error by deleting ten days from that year, and decreed that a centurial year would not be a leap year unless divisible by 400. To date, this reform is the last so far. The program ends with an illustration of another proposed calendar, the World Calendar.

SUGGESTED PRE-VIEWING ACTIVITIES

1. Discuss: – the origin of the names for the months. (i.e. January after the Roman god, Janus.)
   – the origin of the names for the days of the week.
2. The following questions could be researched by groups or individuals then discussed: – Why has January 31 days, April 30 days etc.
   – Why is an extra day required every four years in our calendar?
   – Why has February 28 days today, but the early Roman calendar used 29 days.
   – When will our next centurial leap year occur?

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss the advantages and/or disadvantages of our present day calendar and the proposed new "World Calendar".
2. Have the students construct a Sun Dial of their own design or similar to the ones shown below.



References

LAND, *The Language of Mathematics*, Longmans.

PROGRAM 3

ONE AND ONE AND ONE AND ONE

Long before numeration systems were developed, man made use of his fingers as a computing aid. From this rather simple beginning computing devices have developed considerably up to their present state.

In order to more fully understand the development of these computing devices this program will investigate some of their early history.

PROGRAM CONTENT

By using his arms and fingers in various positions, early man was able to represent numbers larger than ten. It is believed the Chinese were one of the earliest peoples to use counting rods for computations. These counting rods were grouped in triangular patterns, which were then grouped to form hexagonal bundles. Another aid for computation that developed was the abacus. Grooves to represent ones, tens, hundreds, etc., were drawn in the sand, but later marble was used. Stones were placed in these grooves to represent numbers. Grooves to represent the intermediate values of 5, 50, 500 etc. were made and fewer stones would then be necessary to do computations. The Roman abacus built with slots, and the Chinese sliding-bead abacus are shown. In many parts of the world, abaci are still used for computations.

SUGGESTED PRE-VIEWING ACTIVITIES

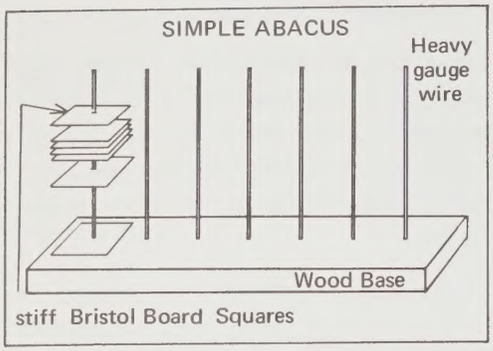
1. Discuss how the fingers and arms held in various positions could be used to represent numbers greater than ten.
2. Discuss other primitive computing methods which could have possibly been used.
3. Carry on some simple transactions using a primitive computing method.

SUGGESTED POST-VIEWING ACTIVITIES

1. The students could be encouraged:

– to build a simple abacus using sand, or a Chinese sliding-bead abacus,

– to do simple arithmetical operations using an abacus.



2. Examine contemporary situations where automatic counting is necessary e.g. turnstiles, printing, machines, counters on tape recorders, odometers, etc.

3. Discuss the following from the program:

   – the use of abaci for computations in some parts of the world.

   – the origin of the word abacus, calculation.

   – the use of abaci in banks.

References

RIPLEY AND TAIT, *Enrichment Mathematics,* Copp Clark.

SMITH, D.E., *History of Mathematics,* Ginn and Company.

PROGRAM 4

ABACUS TO COMPUTER

The purpose of this program is to show that the advent of modern computers and computer science is the result of many years of creative thought and experimentation.

PROGRAM CONTENT

This program traces the development of mechanical aids for computations from the abacus to the computer. In 1642, Blaise Pascal, at the age of 19, developed an adding machine which incorporated automatic carrying in arithmetical operations. The contributions of Gottfried Leibniz, Matteus Hahn, and Charles Babbage are then discussed, and illustrated. With improved technology, calculating machines have progressed to the stage of the compact electrically-powered ones we have today. The computer is the highest achievement reached by man in his efforts to speed up the process of computation. The use of the computer in various phases of transportation, business and science is shown.

SUGGESTED PRE-VIEWING ACTIVITIES

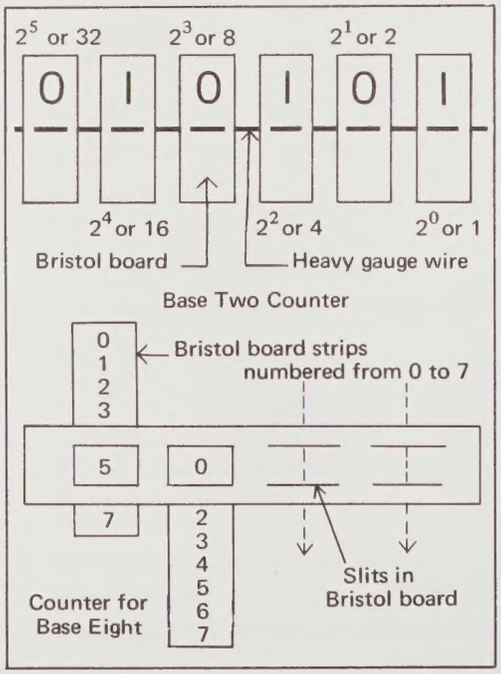
1. Research assignments could be assigned to prepare for the discussion the contributions of Blaise Pascal, Gottfried Leibniz, Matteus Hahn, Charles Babbage, to name a few, in the development of mechanical aids for computation.

2. Discuss the various areas in which calculators are used, e.g. supermarkets, banks, etc.

3. The areas in which computers are used could be discussed, e.g. street traffic, railways, airports, telephone communications, laboratories, space technology, etc.

SUGGESTED POST-VIEWING ACTIVITIES

1. The students could be encouraged to build a simple calculating device.



2. The various types of punched cards used for telephone, gasoline, credit charges could be examined. The following questions could be considered:

   – Why are holes necessary in the cards?

   – What advantage and/or disadvantage may exist for the company that uses them?

References

KAHL, *Computer in Business,* Gage

LOUIS, F.B., *Computers 1, 2.,* MacMillan

RIPLEY AND TAIT, *Enrichment Mathematics, Copp Clark*

THOMAS, S., *Computers,* Holt Rinehart & Winston.

PROGRAM 5

THE HISTORY OF NUMERALS

Just as there were great differences in language throughout the ancient world there were also many variations in recording numbers. The methods and materials used reflected the environment of the people.

The purpose of this program is to investigate the early development of numerals.

PROGRAM CONTENT

The program outlines the development of numerals from the earliest forms to the Hindu-Arabic numerals. Man's first method of recording numbers was probably a tally system using objects such as stones. The ancient Babylonians and Sumerians evolved a method where wedge marks were used in soft clay, which was then baked to obtain a permanent record. The symbols used by the Mayans and Egyptians are then shown. The program then illustrates the numerals written by the Greeks and Romans. The origins and development of the Hindu-Arabic numerals 0, 1, . . . , 9 are then described. Although the Hindu-Arabic system of numerals is generally accepted today, other systems are still in use, e.g. the Chinese numeral system.

SUGGESTED PRE-VIEWING ACTIVITIES

1. Group research assignments are suggested for the following numeration systems; Babylonian, Sumerian wedge marks, Mayan, Aztec, Egyptian, Greek, Roman.

2. The students could write numbers using the different systems and do simple calculations with them.

SUGGESTED POST-VIEWING ACTIVITIES

1. Trace the development of the forms and the origins of the Hindu-Arabic numerals 0, 1, 2, . . . 9. Why are these numbers referred to as the Hindu-Arabic system?

2. Examine what other systems are used today.

3. Discuss the following:
   - Numbers written on some cheques use a special ink which a computer can interpret.
   - Counting with various combinations of hand and finger formations is still done.

   (c) The Hindu-Arabic system of numerals did not gain fast acceptance in the Roman Empire.

   (d) Leonardo da Pisa played a considerable part in making the advantages of the Hindu-Arabic system more widely known.

References

EVES, *An Introduction to the History of Mathematics,* Holt Rinehart & Winston.

LIFE SCIENCE LIBRARY, *Mathematics.*

PROGRAM 6

THE HISTORY OF INDIAN MATHEMATICS

This program stresses the universal nature of the development of mathematics by illustrating the contributions made to mathematical thought by various people from all parts of the world. It also shows how early mathematics developed as part of the tradition and pseudo-science of the time.

PROGRAM CONTENT

The program opens with a brief note on the contributions of a number of mathematicians. The three periods of Indian Mathematics are then examined: Sulvasutra, ca. 200 AD; Astronomical, ca. 400-600 AD and Hindu, ca. 600-1200 AD. In the first period, the early development of mathematics is traced. Aryabhata was a mathematician of the second era and his written work influenced the mathematics of the Hindu period. One outstanding mathematician of this period was Brahmagupta who wrote a 21-chapter book on various topics of mathematics. A problem found in the work of Mahavira the Learned illustrates his fanciful approach to the study of mathematics. e.g. "How many camels were there in this herd if a quarter of a herd of camels is seen in the forest, twice the square root of the herd goes to the slopes, and two times two camels are found on a river bank?" The work of the 19th century Indian mathematician, Ramanujan, is illustrated. He determined the total number of ways a given natural number could be expressed as a sum of smaller whole numbers.

SUGGESTED PRE-VIEWING ACTIVITIES

1. Examine briefly some of the contributions of the mathematicians Pythagoras, Euclid, Newton, Descartes, Leibniz, Einstein.

2. Discuss the properties of zero with respect to addition, subtraction, multiplication and division.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss the problem of Mahavira the Learned. (If x represents the number of camels, the equation becomes: $\frac{1}{4}x + 2\sqrt{x} + 4 = x$).

2. Discuss the following as noted in the program:
   - the problem of squaring the circle.
   - the volume V of a pyramid in terms of the area "A" of the base and the height "h". ($V = \frac{1}{3}Ah$)

   $\pi \doteq 3\frac{177}{1250}$ or (3.1416)

   Pythagoras theorem.

   (e) the area of a circle is equal to that of a rectangle having a width equal to the radius, and the length equal to half of the circumference.

Reference

EVES, *An Introduction to the History of Mathematics,* Holt, Rinehart & Winston.

PROGRAM 7

SHAPES OF NUMBERS

The aim of this program is to remind the viewer that there is still much to be discovered in mathematics. The more he learns about it the more avenues he opens for investigation. The classification of whole numbers according to their geometric configuration is a simple example of the possibilities open to all.

PROGRAM CONTENT

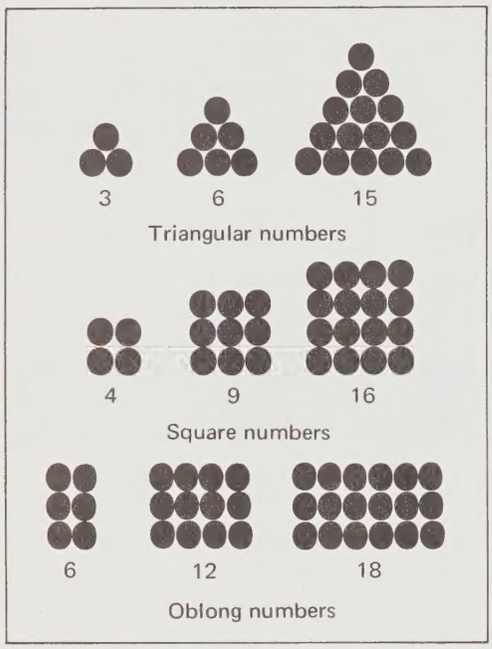
The natural numbers are represented by geometrical configurations of dots. Triangular numbers (so called because they can be represented as a triangular array of dots) were studied by the German mathematician, Carl Gauss - who recognized that each sum of consecutive natural numbers represented a triangular number. Other numbers illustrated on the program are oblong, pentagonal and hexagonal. The numbers found by adding consecutive triangular numbers are represented in the form of tetrahedrons, and thus called tetrahedral numbers. In a similar way, pyramidal numbers are formed. The program ends with two examples of magic squares.

SUGGESTED PRE-VIEWING ACTIVITIES

1. The following could be discussed:
   - When consecutive natural numbers are added, the sums obtained represent the triangular numbers 1, 3, 6, 10, ...
   - When consecutive odd numbers are added, the sums obtained represent

the square numbers 1, 4, 9, 16, . . .

– When consecutive even numbers are added, the sums obtained represent the oblong numbers 2, 6, 12, 20, . . .



SUGGESTED POST-VIEWING ACTIVITIES

1. Determine the triangular numbers by using the expression ½n (n+1) for the nth one. (In this way, Gauss was able to determine that the sum 1 + 2 + 3 + 4 + . . . +100 represented the 100th triangular number, and easily obtained the value 5050).

2. By using geometrical configurations, the students could be encouraged to show that:
   – each square number is the sum of two consecutive triangular numbers.
   – each oblong number is the sum of two equal triangular numbers.

3. Discuss the following:
   – Consecutive sums of the numbers 1, 4, 7, 10, . . . are referred to as the pentagonal numbers 5, 12, 22, ...
   – Consecutive sums of the numbers 1, 5, 9, 13, . . . are referred to as the hexagonal numbers 6, 15, 28, ...

4. The geometrical configuration represented by the following numbers could be examined:
   – tetrahedral numbers – addition of consecutive triangular numbers.
   – pyramidal numbers – addition of consecutive square numbers.
   – cubic numbers – addition by the following pattern 1, 3+5, 7+9+11, 13+15+17+19, ...

5. A magic square is given below for purposes of discussion.

6. Man has named numbers in many ways. The following could be discussed: perfect numbers, deficient numbers, abundant numbers, Pythagorean numbers, friendly numbers, unlucky numbers, to name a few.

8	1	6
3	5	7
4	9	2

Reference

EVES, *An Introduction to the History of Mathematics,* Holt, Rinehart & Winston.

MATHEMATICS – Intermediate Division
ADVENTURES IN MATH
7 programs, 10 minutes each

Prog. No.	CBC 9:50-10:00	CBLT 10:00-10:10	CHCH 9:30-9:40	CKCO 9:50-10:00	CJOH 10:00-10:10	CBOT 11:15-11:25	CKLW 10:20-10:30
1	OCT. 22 – 11:15-11:25 DEC. 10 – 11:15-11:25 JAN. 9	MAR. 24	DEC. 2	APR. 8 – 10:00-10:10	FEB. 27	OCT. 29 DEC. 10	APR. 15 – 10:30-10:40
	MAR. 24 – 10:05-10:15 MAY 17 – 10:00-10:10					MAY 17 – 10:00-10:10	MAY 17 – 10:00-10:10
2	OCT. 29 – 11:15-11:25 DEC. 17 – 11:15-11:25 JAN. 26 MAR. 26 – 10:20-10:30 MAY 24 – 10:00-10:10	MAR. 31	DEC. 5	APR. 22	MAR. 3 – 9:10-9:20	OCT. 29 DEC. 17 MAY 24 – 10:00-10:10	APR. 29 MAY 24 – 10:00-10:10
3	NOV. 5 – 11:15-11:25 NOV. 6 – 11:15-11:25 JAN. 23 APR. 2 – 10:20-10:30 MAY 31 – 10:00-10:10	APR. 25 – 10:20-10:30	DEC. 10 – 9:55-10:05	APR. 25	MAR. 6	NOV. 5 MAY 31 – 10:00-10:10 NOV. 6	MAY 2 NOV. 6 – 11:15-11:25 MAY 31 –
4	NOV. 12 – 11:15-11:25 NOV. 13 – 11:15-11:25 JAN. 30 APR. 14 – 10:20-10:30		DEC. 17 – 9:55-10:05	APR. 29	MAR. 13	NOV. 12 NOV. 13	MAY 6 NOV. 13 – 11:15-11:25
5	NOV. 19 – 11:15-11:25 MAR. 5 – 9:30-9:40		DEC. 19	MAY 2	MAR. 27	NOV. 19	MAY 9
6	NOV. 26 – 11:15-11:25 MAR. 10 – 10:20-10:30			MAY 6	MAY 2 – 9:55-10:05	NOV. 26	MAY 13
7	DEC. 3 – 11:15-11:25 MAR. 12 – 10:05-10:15			MAY 13	MAY 9 – 9:55-10:05	DEC. 3	MAY 20

ADVENTURES IN MATHEMATICS